

Transition Distribution Amplitudes

J.P. Lansberg^{1*}, B. Pire¹ and L. Szymanowski^{2,3}

¹*Centre de Physique Théorique, École Polytechnique, CNRS, 91128, Palaiseau, France*

²*Fundamental Interactions in Physics and Astrophysics, Université de Liège, Belgique*

³*Soltan Institute for Nuclear Studies, Warsaw, Poland*

We give a brief overview of the theoretical status of the Transition Distribution Amplitudes and discuss their experimental near future.

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1. Introduction

According to a now well-established framework,¹ the Bjorken limit of near forward exclusive reactions with a hard probe allows factorisation of the leading twist amplitudes into a perturbatively calculable sub-process at quark and gluon level on the one hand and hadronic matrix elements of light-cone non-local operators describing the transition from the baryon target to a final baryon on the other hand. Those matrix elements can be expressed through hadron distribution amplitudes (DAs) and generalised parton distributions (GPDs). Experimental data² from DESY and JLab are now confirming this framework, and they seem to show that its applicability is quite precocious in terms of Q^2 . The harmonic analysis³ of spin asymmetries which are particularly sensitive to the interference between the deeply-virtual-Compton scattering (DVCS) and Bethe-Heitler processes is very relevant for that purpose.

Investigations on GPDs are very important since they are new QCD objects which carry much information on the hadronic structure. A further generalisation of the GPD concept has been proposed⁴ in cases where the initial and final states are different hadronic states. If those new hadronic objects are defined through a quark-antiquark operator (meson to meson or meson to photon transition), we call them *mesonic* transition distribution amplitudes (TDA),⁵ if they are defined through a three

*lansberg@cpht.polytechnique.fr

quark operator (baryon to meson or baryon to photon transition), we call them *baryonic* transition distribution amplitudes.⁶

Both appear in the description of hard exclusive processes, where a highly off-shell photon provides a hard scale permitting, on the one hand, to treat perturbatively the interaction between the photon and the quarks off the target, on the other hand to advocate the factorisation of the amplitude as a convolution of a hard amplitude M_h with the *universal* TDAs describing the non-perturbative transition between two hadronic states and DAs describing the formation of another hadron. This is illustrated in Fig. 1 for three cases. The first is the process $\gamma^*\gamma \rightarrow A\pi$ at small t , where A is a meson. There appear the mesonic $\gamma \rightarrow \pi$ TDAs. The second

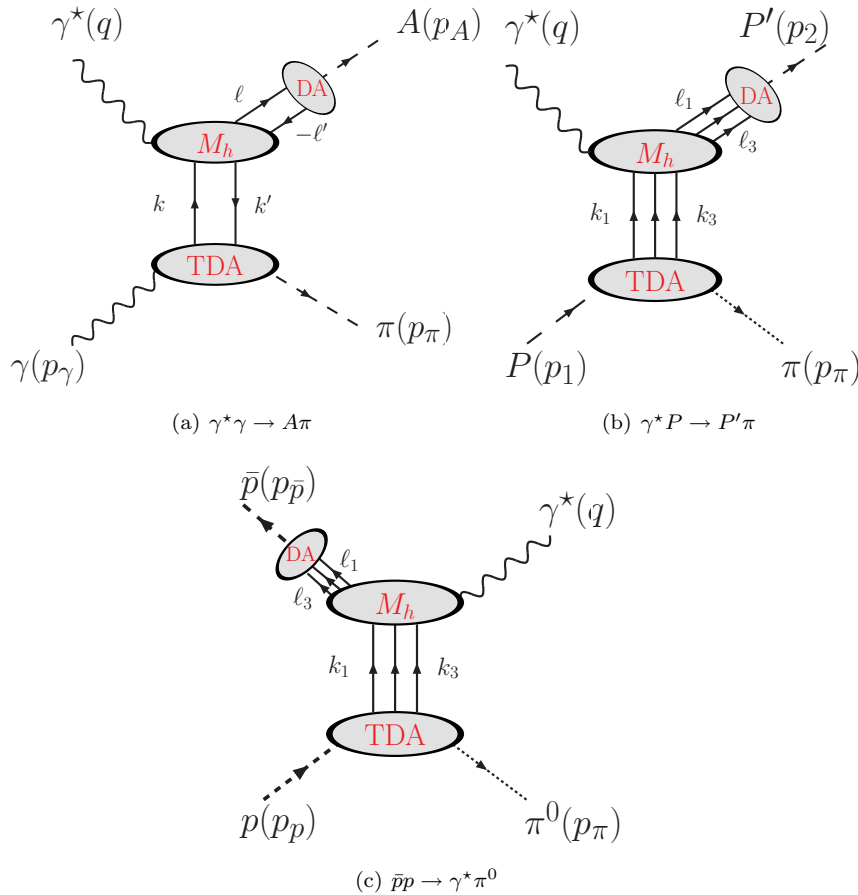


Fig. 1. (a) Illustration of the factorisation for $\gamma^*\gamma \rightarrow A\pi$ at small transfer momentum. (b) Idem for backward electroproduction of a pion. (c) Idem for $\bar{p}p \rightarrow \gamma^*\pi^0$.

is the backward electroproduction of a pion and the third is a crossed process $\bar{p}p \rightarrow \gamma^* \pi^0$. In those two last processes, there appear the baryonic $p \rightarrow \pi^0$ TDAs.

2. The mesonic TDAs

2.1. Mesonic TDAs vs. GPDs

For definiteness, let us consider the $\gamma \rightarrow \pi^-$ TDAs. Those are defined from the correlators^{a b}

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^-(p_{\pi^-}) | \bar{d}(-\frac{z}{2})(\Gamma)u(\frac{z}{2}) | \gamma(p_\gamma, \varepsilon) \rangle \Big|_{z^+=0, z_T=0}, \quad (1)$$

where Γ is one of the Dirac forms, $\gamma^\mu, \gamma^\mu \gamma^5$ or $\sigma^{\mu\nu}$. The first two enable to respectively define the vector TDA $V^{\gamma\pi^-}(x, \xi, \Delta^2)$ and the axial-vector one $A^{\gamma\pi^-}(x, \xi, \Delta^2)$, going along with their respective Lorentz structures.⁵ The latter Dirac form, $\sigma^{\mu\nu}$, is related to two chiral-odd tensorial TDAs $T_1^{\gamma\pi^-}(x, \xi, \Delta^2)$ and $T_2^{\gamma\pi^-}(x, \xi, \Delta^2)$.

The mesonic TDAs have much in common with meson GPDs. They are defined from matrix elements of the same quark - antiquark operator and thus obey the same QCD evolution equations. Sum rules may be derived for the photon to meson TDAs. Since the local matrix elements appear in radiative weak decays, we can relate the TDAs to the vector and axial form factors F_V and F_A in the π^\pm case and to $F_{\pi^0\gamma^*\gamma}$ in the π^0 case. Those form factors are well measured.⁷

Moreover, the TDAs satisfy similar polynomiality conditions and they may be constructed from a spectral decomposition, in analogy with the construction of GPDs through double distributions.⁸ The x and ξ dependence of the TDAs is then given as

$$\int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi\alpha) f(\beta, \alpha).$$

In the GPD case, $f(\beta, \alpha) = q(\beta)h(\beta, \alpha)$ with $q(\beta)$ the forward quark distribution and $h(\beta, \alpha)$ a profile function. Note however that TDAs possess different properties than GPDs with respect to time reversal since initial and final states are different. A consequence of this is the appearance of odd-powers of ξ in their moments in x .

^aWorking in light-like (or axial) gauge both for the gluons and the photons, we omitted to include the QCD and QED Wilson line between the quark fields.

^bIf p and p' are respectively the momenta of the initial and final particles in the TDAs, we have $(p' - p) = \Delta$, $t = \Delta^2$, $P = 1/2(p + p')$ and $2\xi = -\Delta^+/P^+$. We also have $\xi \approx x_B/(2 - x_B)$.

In the aforementioned case of $\gamma \rightarrow \pi^-$, the leading-twist decomposition of the matrix element differs from the π^- GPD one mainly because of the presence of the photon polarisation vectors $\varepsilon(p_\gamma)$ – a similar situation would occur for the $\pi \rightarrow \rho$ transition. This enables to build 4 independent leading-twist Lorentz structures, and thus to define 4 TDAs, whereas there are only 2 leading-twist pion GPDs.

2.2. A perturbative limit

An interesting perturbative limit of a mesonic TDA may be derived⁹ by considering the Born order amplitude \mathcal{A} for the process

$$\gamma_L^*(q') \gamma_L^*(q) \rightarrow \rho_L^0(k') \rho_L^0(k).$$

in the region where $Q'^2 \gg Q^2$ and in the forward kinematics ($t = t_{min}$).

One can see that the amplitude factorises and the $\gamma_L^* \rightarrow \rho_L$ vector $V(x, \xi, t)$ TDA^c is obtained at leading order as

$$V(x, \xi, t_{min}) = 3[\Theta(1 \geq x \geq \xi)\Phi(\frac{x - \xi}{1 - \xi}) - \Theta(-\xi \geq x \geq -1)\Phi(\frac{1 + x}{1 - \xi})],$$

where Φ is the ρ DA. This TDA describes the transition between the *least off-shell* photon (Q^2) and the longitudinally polarised ρ to which it is collinear. Note that this perturbative expression vanishes in the ERBL region.

2.3. Models

We have already at our disposal several models for the vector and axial-vector TDAs describing the pion to photon transition. In Refs 10,11, those were modelled via double-distributions and more recently in the spectral quark model.¹² Other approaches¹³ used to construct pion GPDs could provide us with reasonable modelling of the mesonic TDAs. Let us cite for instance the Nambu-Jona Lasinio Model.¹⁴ Finally, lattice QCD, which has been recently applied to extract moments of pion GPDs,¹⁵ could also be applied to the TDA case.

^c Defining n_1 and n_2 such that $q = \frac{-Q^2}{(1+\xi)s}n_1 + (1+\xi)n_2$ and $k = (1-\xi)n_2$ and considering the quark antiquark operator along the light-cone vector $n = 2n_1/s$, $s = 2n_1 \cdot n_2$, $V(x, \xi, t)$ is defined as (omitting the QED Wilson line)

$$\int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \langle \rho_L^q(k) | \bar{q}(-\frac{z}{2}) \not{n} q(\frac{z}{2}) | \gamma^*(q) \rangle = \frac{eQ_q \epsilon_\nu(q)}{P+Q^2} [(1+\xi)n_2^\nu + \frac{Q^2 n_1^\nu}{s(1+\xi)}] V(x, \xi, t).$$

2.4. *Experimental possibilities*

The introduction of the $\gamma \rightarrow \text{meson}$ TDAs completes the kinematical domain of understanding of the reactions $\gamma\gamma^* \rightarrow M_1 M_2$ in the framework of QCD factorisation, supplementing the near threshold kinematical domain described by generalised distribution amplitudes (GDAs)¹⁶ and the fixed (or large) angle domain historically described by the Brodsky-Lepage factorisation.¹⁷ Data have been collected at LEP and CLEO on these reactions, mostly in the GDA domain for $\rho\rho$ final states, with some phenomenological success.¹⁸ More data are obviously needed and are eagerly waited for in the TDA region, and much hope comes from the high luminosity electron colliders. This requires in general the exclusive detection of two mesons, when tagging simultaneously one outgoing electron.

Another way to access the mesonic TDAs is to study DVCS on virtual pion target, which may be studied¹⁹ at Hermes and JLab in the reaction $\gamma^* p \rightarrow \gamma \pi^+ n$, when the transition $p \rightarrow n$ is dominated by the pion pole and the π^+ flies in the direction of the γ^* in the $\gamma\pi^+$ CMS.

3. The baryonic TDAs

3.1. *Definitions*

Let us concentrate here on the $p \rightarrow \pi$ TDAs. The leading twist TDAs for the $p \rightarrow \pi^0$ transition are defined from the correlator :

$$\langle \pi^0(p_\pi) | \epsilon^{ijk} u_\alpha^i(z_1 n) u_\beta^j(z_2 n) d_\gamma^k(z_3 n) | p(p_p, s_p) \rangle$$

These TDAs are matrix elements of the same operator that appears in baryonic distribution amplitudes. The known evolution equations of this operator lead to derive evolution equations which have different forms in different regions ; one defines one ERBL and two DGLAP regions much in the same spirit as in the GPD case, so that the evolution equations in momentum space depend on the signs of the quark momentum fractions x_i .

As for DAs, an asymptotic solution for this evolution equation exists but the phenomenological study of electromagnetic form factors leads us to strongly doubt that it is of any phenomenological relevance. In some sense, this is not a surprise since the corresponding asymptotic solution ($\delta(x)$) for parton distribution functions is far from a realistic description of DIS data. We thus do not propose to take an asymptotic TDA as a realistic input for phenomenology.

On the other hand, there exists an interesting soft limit²⁰ when the emerging pion momentum is small, which allows to relate proton \rightarrow pion TDAs to proton DAs. The well-known soft pion theorems indeed allow to write:

$$\langle \pi^a(p_\pi) | \mathcal{O} | P(p_1, s_1) \rangle \rightarrow -\frac{i}{f_\pi} \langle 0 | [Q_5^a, \mathcal{O}] | P(p, s) \rangle$$

when $\xi \rightarrow 1$ ($E_\pi \rightarrow 0$) ; the neglected nucleon pole term, which does not contribute at threshold but is likely to be important for ξ significantly different from 1, may also be taken into account. One then gets relations between the nucleon DAs²¹ A^p , V^p and T^p on the one hand and the $p \rightarrow \pi$ TDAs $V_1^{p\pi^0}$, $A_1^{p\pi^0}$ and $T_1^{p\pi^0}$ on the other hand :

$$\begin{aligned} V_1^{p\pi^0}(x_1, x_2, x_3, 1, M^2) &= \frac{1}{4} V^p\left(\frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2}\right), \\ A_1^{p\pi^0}(x_1, x_2, x_3, 1, M^2) &= \frac{1}{4} A^p\left(\frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2}\right), \\ T_1^{p\pi^0}(x_1, x_2, x_3, 1, M^2) &= \frac{3}{4} T^p\left(\frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2}\right). \end{aligned}$$

To conclude, let us mention that the proton to photon TDAs, entering the description of backward DVCS, have been defined in Ref. 22.

3.2. *Experimental situation*

As we mentioned above, $p \rightarrow \pi$ baryonic TDAs appear in the description of backward electroproduction of a pion on a proton target. In terms of angle, in the γ^*p center of momentum (CM) frame, the angle between the γ^* and the pion, θ_π^* , is close to 180° . We then have $|u| \ll s$ and $t \simeq -(s + Q^2)$, in contrast to the fixed angle regime $u \simeq t \simeq -(s + Q^2)/2$ ($\theta_\pi^* \simeq 90^\circ$) and the forward (GPD) one $|t| \ll s$ and $u \simeq -(s + Q^2)$ ($\theta_\pi^* \simeq 0^\circ$).

The TDAs appear also in similar electroproduction processes such as $ep \rightarrow e(p, \Delta^+)(\eta, \rho^0)$, $ep \rightarrow e(n, \Delta)(\pi^+, \rho^+)$, $ep \rightarrow e\Delta^{++}(\pi^-, \rho^-)$. Those processes have already been analysed, at backward angles, at JLab in the resonance region, *i.e.* $\sqrt{s_{\gamma^*p}} = W < 1.8$ GeV, in order to study the baryonic transition form factors in the π channel²³ or in the η channel.^{24,25} Data are being extracted in some channels above the resonance region. The number of events seems large enough to expect to get cross section measurements for $\Delta_T^2 < 1$ GeV², which is the region described in terms of TDAs. Hermes analysis²⁶ for forward electroproduction may also be extended to larger values of $-t$. It has to be noted though that present studies are limited to Q^2 of order a few GeV², which gives no guarantee

to reach the TDA regime yet. Higher- Q^2 data may be obtained at JLab-12 GeV and in muoproduction at Compass within the next few years. Besides comparisons with forthcoming experimental data, one may also consider results from global Partial Wave Analysis (*e.g.* SAID²⁷).

Crossed reactions involving TDAs in proton-antiproton annihilation (GSI-FAIR²⁸), with time-like photons (*i.e.* di-leptons) can also be studied with other mesons than a pion, *e.g.* $\bar{p}p \rightarrow \gamma^* (\eta, \rho^0)$, or on a different target than proton $\bar{p}N \rightarrow \gamma^* \pi$. Finally, one may also consider associated J/ψ production with a pion $\bar{p}p \rightarrow \psi \pi^0$ or another meson $\bar{p}p \rightarrow \psi (\eta, \rho^0)$, which involve the *same* TDAs as with an off-shell photon or in backward electroproduction. They will serve as very strong tests of the universality of the TDAs in different processes.

3.3. Models and applications

The first application of baryonic TDAs was centered on backward electroproduction of a pion.²⁰ In that case, the hard contribution which consists in the scattering of the hard photon with three quarks is known at leading order. Extrapolating the limiting value of the TDAs obtained from the soft pion theorems to the large- ξ region, we obtained a first evaluation of the unpolarised cross section for backward electroproduction. This estimate, which is unfortunately reliable only in a restricted kinematical domain (large- ξ), shows an interesting sensitivity to the underlying model for the proton DA. This study will soon be extended to hard exclusive production of a $\gamma^* \pi^0$ pair in $\bar{p}p$ annihilation at GSI-FAIR.²⁹

In the near future, a modelling inspired from the soft-pion limit including the nucleon pole term will be applied to backward electroproduction at $\theta_\pi^* \neq 180^\circ$. Application of the Pion-cloud Model³⁰ to $p \rightarrow \pi$ transitions are also expected to be available soon.³¹ Specific non-perturbative approaches such as lattice studies, instanton-based models, chiral perturbation theory,³² AdS-CFT correspondence,³³ can also provide us with realistic expression for the TDAs to be checked experimentally and are therefore eagerly waited for.

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